WorkBook

BINOMIAL EXPANSIONS

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Distributive Property

Multiplication distributes over addition. That is, the product of a sum is the sum of the products.

<u>Example 1</u>

Solution	Explanations
$2(3d + 4)$	Each term inside the brackets is multiplied by the number outside the brackets. **Note**. Since there is no sign between the 2 and the bracket, it is a multiplication. 
$= \ 6d + 8$	$2 \times 3d = 6d \text{ and } 2 \times 4 = 8$

<u>Example 2</u>

Solution	Explanations
$m(4m - 5)$	Each term inside the brackets is multiplied by the number outside the brackets. **Note**. Since there is no sign between the *m* and the bracket, it is a multiplication. 
$= \ 4m^2 - 5m$	$m \times 4m = 4m^2 \text{ and } m \times -5 = -5m$ **Note** The minus sign between the two terms in the expression results from the second product being negative.

<u>Example 3</u>

Solution	Explanations
$2a(3a - 4d)$	Each term inside the brackets is multiplied by the number outside the brackets. **Note**. Since there is no sign between the *m* and the bracket, it is a multiplication. 
$= \ 6a^2 - 8ad$	$2a \times 3a = 6a^2 \text{ and } 2a \times -4d = -8ad$

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Difference of 2 squares

A product of conjugates will expand to a Difference of Two Squares.
Conjugates are binomial expressions that only by the sign separating the two terms in the binomial expressions.

Eg. $(u - 8)$ *and* $(u + 8)$ are conjugates
$(8u - k)$ and $(8u + k)$ are conjugates

Square the first – Square the second

Example 1

Solution	Explanations
$(u - 8)(u + 8)$	Square the first – Square the second. $u^2 = (u)^2$ and $64 = (8)^2$ i.e. $(u)^2 – (8)^2 = u^2 - 64$
$= u^2 - 64$	

Example 2

Solution	Explanations
$(5k - 6v)(5k + 6v)$	Square the first – Square the second. $(k)^2 = k^2$ and $(5)^2 = 25$ $(v)^2 = v^2$ and $(6)^2 = 36$ i.e. $(5)^2(k)^2 – (6)^2(v)^2 = 25k^2 - 36v^2$
$= 25k^2 - 36v^2$	

Perfect Squares
Perfect squares result from the squaring of a binomial expression.

Eg. $(t + 5)^2 = t^2 + 10t + 25$

Square the first + Double the Product + Square the second

Example 1

Solution	Explanations
$(h + 8)^2$	Square the first + Double the Product + Square the second. $(h)^2 = h^2 \; + \; 2 \times h \times 8 = 16h$ and $(8)^2 = 64$
$= h^2 + 16h + 64$	

Example 2

Solution	Explanations
$(4a - 5s)^2$	Square the first + Double the Product + Square the second. $16 = (4)^2 = 16$ and $(a)^2 = a^2$ and $2 \times 4a \times -5s = -40as$ and $(5)^2 = 25$ and $(s)^2 = s^2$ *i.e.* $(4a)^2 + (2 \times 4a \times -5s) + (5s)^2$
$= 16a^2 - 40as + 25s^2$	

Other binomial expansions
Quadratic trinomials are either monic or non-monic. The Binomial expansion leading to them are carried out differently.
Note:

Monic means one	$b^2 - 21b + 108$	(the coefficient of b^2 is 1)
	$m + 3$	(the coefficient of m is 1)
Non-monic means "not one"	$40k^2 - 57k + 20j^2$	(the coefficient of k^2 is 40)
	$4d + 3$	(the coefficient of d is 4)
	$1 - x$	(the coefficient of x is -1)

Monic

Monic binomial expansions can be expanded by repetition of the distributive property.

Example 1

Solution	Explanations
$(b - 9)(b - 12)$	$= (b - 9)(b - 12)$ $= b(b - 12) - 9(b - 12)$
$= b^2 - 12b - 9b + 108$	Collect the like terms (simplify)
$= b^2 - 21b + 108$	

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An alternate ‘in your head’ method.

Square the pronumeral + Sum of constants × pronumeral + Product of constants

<u>Example 1</u>

Solution	Explanations
$(b - 9)(b - 12)$	Square the pronumeral + Sum of constants × pronumeral + Product of constants $(b)^2 = b^2$ <u>and</u> $(-9 + -12) \times b = -21b$ and $(-9 \times -12) = 108$
$= \ b^2 - 21b + 108$	

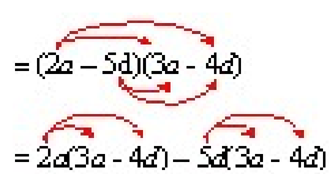
<u>Example 2</u>

Solution	Explanations
$(m + 5)(m - 8)$	Square the pronumeral + Sum of constants × pronumeral + Product of constants $(m)^2 = m^2$ <u>and</u> $(5 + -8) \times m = -3m$ and $(5 \times -8) = -40$
$= \ m^2 - 3m - 40$	

Non-Monic trinomials

Non-monic binomial expansions are expanded by repetition of the distributive property.

<u>Example 1</u>

Solution	Explanations
$(2a – 5\text{d})(3a - 4d)$	
$= \ 6a^2 – 8ad – 15ad + 20d^2$	Collect the like terms (simplify)
$= \ 6a^2 – 23ad + 20d^2$	

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Multiple Steps

Some expansions can have more than two parts. Also, there are trinomial expansions, etc.

Example 1

Solution	Explanations
$2(m + 3)(m + 12)$	Multiply the two binomials
$= 2(m^2 + 15m + 36)$	Multiply the product by 2
$= 2m^2 + 30m + 72$	

Example 2

Solution	Explanations
$(m + 3)(m + 12)(m - 1)$	Multiply the first two binomials
$= (m^2 + 15m + 36)(m - 1)$	Multiply the product by the third binomial
$= m^3 + 15m^2 + 36m - m^2 - 15m - 36$	Collect like terms. (simplify)
$= m^3 + 14m^2 + 21m - 36$	