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SOLVING EQUATIONS

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Solving Equations

OBJECT
The object of solving equations is to find the value of the pronumeral that's makes the equation true.

PROCESS
Equations are solved by performing a series of opposite operations until the pronumeral is alone, equal to a number. These opposite operations must be performed to both sides of the equation.

Example; Solve $3d + 4 = 10$.

Step	Solution	Explanations
	$3d + 4 = 10$	First determine what is happening to the pronumeral. $\Rightarrow$ The pronumeral *d* is multiplied by 3 and then 4 is added. $\Rightarrow$ Reverse this process. ie undo the *plus 4* and then undo the *times by 3*.
1.	$3d + 4^{-4} = 10^{-4}$	The opposite to *plus 4* is *minus 4*. So subtract 4 from both sides. This gives $3d = 6$.
2.	$\frac{3d}{3} = \frac{6}{3}$ $d = 2$	The opposite of *times by 3* is *dividing by 3*. So divide both sides by three. This is best done as a fraction This is the solution to the equation. ie For $3d + 4 = 10$ substitute $d = 2$ $3 \times 2 + 4 = 10$

GLOSSARY

Pronumeral	A letter or symbol used to represent a number.
Expression	A mathematical expression is a combination of numbers, pronumerals and operations.
Operation	A mathematical operation modifies or changes the value of a number or expression
Opposite operation	An opposite operation undoes the changes caused by another operation. Eg subtraction undoes addition, division undoes multiplication, etc **Opposite Operations** addition and subtraction multiplication and division squaring and square root
Equation	Also called a number sentence. It is an algebraic expression containing an equal sign. The left side is equal to the right side.
Addition	Other words that indicate this operation; add, plus, sum, and,
Subtraction	Other words that indicate this operation; subtract, minus, difference, take-away,
Multiplication	Other words that indicate this operation; multiply, times, product,
Division	Other words that indicate this operation; divide, goes into, quotient,

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One Step Equations

One step equations only involve one step to solve the equation, that is, one step to get the pronumeral on the left to equal the number.

It is important to note that one step equations show how to undo each type of operation when performing several steps when solving more complex equations.

Example 1

Step	Solution	Explanations
	$a + 3 = 5$	First determine what is happening to the pronumeral. $\Rightarrow$ The pronumeral a has 3 added to it.
1.	$a + 3^{-3} = 5^{-3}$ $a = 2$	The opposite to *plus 3* is *minus 3*. So subtract 3 from both sides This is the solution to the equation. ie For $a + 3 = 5$ substitute $a = 2$ $2 + 3 = 5$

Example 2

Step	Solution	Explanations
	$b - 4 = 7$	First determine what is happening to the pronumeral. $\Rightarrow$ The pronumeral b has 4 subtracted from it.
1.	$b - 4^{+4} = 7^{+4}$ $b = 11$	The opposite of *minus 4* is *plus 4*. So add 4 to both sides This is the solution to the equation. ie For $b - 4 = 7$ substitute $b = 11$ $11 - 4 = 7$

Example 3

Step	Solution	Explanations
	$2c = 8$	First determine what is happening to the pronumeral. $\Rightarrow$ The pronumeral c is multiplied by 2.
1.	$\frac{2c}{2} = \frac{8}{2}$ $c = 4$	The opposite of *times by 2* is *dividing by 2*. So divide both sides by 2 This is the solution to the equation. ie For $2c = 8$ substitute $c = 4$ $2 \times 4 = 8$

Example 4

Step	Solution	Explanations
	$\frac{d}{3} = 4$	First determine what is happening to the pronumeral. $\Rightarrow$ The pronumeral d divided by 3.
1.	$\frac{^{\times 3}d}{3} = 4^{\times 3}$ $d = 12$	The opposite of *divide by 3* is *times by 3*. So multiply both sides by 3 This is the solution to the equation. ie For $\frac{d}{3} = 4$ substitute $d = 12$ $\frac{12}{3} = 4$

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Two Step Equations

Two step equations involve two steps to solve the equation, that is, two steps to get the pronumeral on the left to equal the number.

Two step equations come in many forms. However, the process is still the same.

Example 5

Step	Solution	Explanations
	$2a + 3 = 7$	First determine what is happening to the pronumeral. $\Rightarrow$ The pronumeral a is multiplied by 2 and then 3 is added. $\Rightarrow$ Reverse this process. ie undo the *plus 3* and then undo the *times by 2*.
1.	$2a + 3^{-3} = 7^{-3}$	The opposite to *plus 3* is *minus 3*. So subtract 3 from both sides. This gives $2a = 4$.
2.	$\frac{2a}{2} = \frac{4}{2}$	The opposite of *times by 2* is *dividing by 2*. So divide both sides by two. This is best done as a fraction
	$a = 2$	This is the solution to the equation. ie For $2a + 3 = 7$ substitute $a = 2$ $2 \times 2 + 3 = 7$

Example 6

Step	Solution	Explanations
	$4b - 1 = 11$	First determine what is happening to the pronumeral. $\Rightarrow$ The pronumeral b is multiplied by 4 and then 1 is subtracted. $\Rightarrow$ Reverse this process. ie undo the *minus 1* and then undo the *times by 4*.
1.	$4b - 1^{+1} = 11^{+1}$	The opposite to *minus 1* is *plus 1*. So add 1 to both sides. This gives $4b = 12$.
2.	$\frac{4b}{4} = \frac{12}{4}$	The opposite of *times by 4* is *dividing by 4*. So divide both sides by four. This is best done as a fraction
	$b = 3$	This is the solution to the equation. ie For $4b - 1 = 11$ substitute $b = 3$ $4 \times 3 - 1 = 11$

Example 7

Step	Solution	Explanations
	$7 - c = 10$	First determine what is happening to the pronumeral. $\Rightarrow$ The pronumeral c is subtracted from 7. $\Rightarrow$ Reverse this process. ie undo the *plus 7* and then undo the *times by -1*. **Note** $-c = -1 \times c$
1.	$7^{-7} - c = 10^{-7}$	The opposite to *plus 7* is *minus 7*. So subtract 7 from both sides. This gives $-c = 3$.
2.	$\frac{-c}{-1} = \frac{3}{-1}$	The opposite of *times by -1* is *dividing by -1*. So divide both sides by negative one. This is best done as a fraction
	$c = -3$	This is the solution to the equation. ie For $7 - c = 10$ substitute $c = -3$ $7 - -3 = 10$

Example 8

Step	Solution	Explanations
	$\frac{3d}{2} = 9$	First determine what is happening to the pronumeral. $\Rightarrow$ The pronumeral d is multiplied by 3 then divided by 2. $\Rightarrow$ Reverse this process. ie undo the *divide by 2* and then undo the *times by 3*.
1.	$\frac{^{\times 2}3d}{2} = 9^{\times 2}$	The opposite of *divide by 2* is *times by 2*. So multiply both sides by 2. This gives $3d = 18$
2.	$\frac{3d}{3} = \frac{18}{3}$	The opposite of *times by 3* is *dividing by 3*. So divide both sides by three. This is best done as a fraction
	$d = 6$	This is the solution to the equation. ie For $\frac{3d}{2} = 9$ substitute $d = 6$ $\frac{3 \times 6}{2} = 9$

Example 9

Step	Solution	Explanations
	$5e - 3 = 6$	First determine what is happening to the pronumeral. $\Rightarrow$ The pronumeral e is multiplied by 5 and then 3 is subtracted. $\Rightarrow$ Reverse this process. ie undo the *minus 1* and then undo the *times by 4*.
1.	$5e - 3^{+3} = 6^{+3}$	The opposite to *minus 3* is *plus 3*. So add 3 to both sides. This gives $5e = 9$.
2.	$\frac{5e}{5} = \frac{9}{5}$	The opposite of *times by 5* is *dividing by 5*. So divide both sides by five. This is best done as a fraction
	$b = 1\frac{4}{5}$	This is the solution to the equation. ie For $5e - 3 = 6$ substitute $b = 1\frac{4}{5}$ $5 \times 1\frac{4}{5} - 3 = 6$
It should be remembered that some solutions may be fractions or decimals.		

Example 10

Step	Solution	Explanations
	$5a = 2a + 6$	First determine what is happening to the pronumeral. $\Rightarrow$ The pronumeral a is multiplied by 5 and this is equal to the pronumeral a is multiplied by 2 and then 6 is added. $\Rightarrow$ Reverse this process. ie first get the pronumerals on the left and the undo whatever a is multiplied *by*.
1.	$5a^{-2a} = 2a^{-2a} + 6$	The opposite to *plus 2a* is *minus 2a*. So subtract $2a$ from both sides so all pronumerals are on the left. This gives $3a = 6$.
2.	$\frac{3a}{3} = \frac{6}{3}$	The opposite of *times by 3* is *dividing by 3*. So divide both sides by three. This is best done as a fraction
	$a = 2$	This is the solution to the equation. ie For $5a = 2a + 6$ substitute $a = 2$ $5 \times 2 = 2 \times 2 + 6$ $10 = 10$

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Three Step Equations

Three step equations involve three steps to solve the equation, that is, three steps to get the pronumeral on the left to equal the number.

Three step equations come in many forms. However, the process is still the same. Since the difference in these equations is the additional term with pronumerals, this should be the first part of the equation to be dealt with.

Example 11

Step	Solution	Explanations
	$4a + 2 = 2a + 9$	First determine what is happening to the pronumeral. $\Rightarrow$ The pronumeral a is multiplied by 4 and then 2 is added. This is equal to the pronumeral a is multiplied by 2 and then 9 is added. $\Rightarrow$ Reverse this process. ie first get the pronumerals on the left and getting all the numbers on the right. Then undo whatever a is multiplied *by*.
1.	$4a^{-2a} + 2 = 2a^{-2a} + 9$	The opposite to *plus 2a* is *minus 2a*. So subtract $2a$ from both sides so all pronumerals are on the left. This gives $2a + 2 = 9$.
2.	$2a + 2^{-2} = 9^{-2}$	The opposite to *plus 2* is *minus 2*. So subtract 2 from both sides. This gives $2a = 7$.
3.	$\frac{2a}{2} = \frac{7}{2}$	The opposite of *times by 2* is *dividing by 2*. So divide both sides by two. This is best done as a fraction
	$a = 3\frac{1}{2}$	This is the solution to the equation. ie For $5a = 2a + 6$ substitute $a = 3\frac{1}{2}$ $4 \times 3\frac{1}{2} + 2 = 2 \times 3\frac{1}{2} + 9$ $16 = 16$

Example 12

Step	Solution	Explanations
	$2b + 3 = 6b - 5$	First determine what is happening to the pronumeral. $\Rightarrow$ The pronumeral a is multiplied by 4 and then 2 is added. This is equal to the pronumeral a is multiplied by 2 and then 9 is added. $\Rightarrow$ Reverse this process. ie first get the pronumerals on the left and getting all the numbers on the right. Then undo whatever a is multiplied *by*.
1.	$2b^{-6b} + 3 = 6b^{-6b} - 5$	The opposite to *plus 2a* is *minus 2a*. So subtract $2a$ from both sides so all pronumerals are on the left. This gives $-4b - 3 = -5$.
2.	$-4b + 3^{-3} = -5^{-3}$	The opposite to *plus 2* is *minus 2*. So subtract 2 from both sides. This gives $-4b = -8$.
3.	$\frac{-4b}{-4} = \frac{-8}{-4}$	The opposite of *times by 4* is *dividing by 4*. So divide both sides by four. This is best done as a fraction
	$b = 2$	This is the solution to the equation. ie For $2b + 3 = 6b - 5$ substitute $b = 2$ $2 \times 2 + 3 = 6 \times 2 - 5$ $7 = 7$

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Other Equations

Other equations involve more than three steps to solve the equation, that is, more than three steps to get the pronumeral on the left to equal the number.

These other equations come in many forms. However, the process is still the same. The goal is to get the equation to be a 3-step equation, then 2-step equation, then 1-step equation.

Equations involving brackets

Since the difference in these equations is the brackets, this should be the first part of the equation to be dealt with. There may be different methods, however, this method will simplify the process.

Example 13

Step	Solution	Explanations
	$2(3d + 4) = 20$	First determine what is happening to the pronumeral. $\Rightarrow$ The pronumeral a has 2 added, then this sum is multiplied by 3. $\Rightarrow$ First multiply the brackets. You will notice that this is now a two step equation.
1.	$6d + 8 = 20$	First, multiply the brackets. 
2.	$6d + 8^{-8} = 20^{-8}$	The opposite to *plus 8* is *minus 8*. So subtract 8 from both sides. This gives $6d = 12$.
3.	$\frac{6d}{6} = \frac{12}{6}$	The opposite of *times by 6* is *dividing by 6*. So divide both sides by six. This is best done as a fraction
	$d = 2$	This is the solution to the equation. ie For $2(3d + 4) = 20$ substitute $d = 2$ $2(3 \times 2 + 4) = 20$ $2 \times (10) = 20$ $20 = 20$

Example 12

Step	Solution	Explanations
	$4(2x + 3) = 6x - 4$	First determine what is happening to the pronumeral. $\Rightarrow$ The pronumeral x is multiplied by 2 and then 3 is added. This result is then multiplied by 4. This is equal to the pronumeral x multiplied by 6 and then 4 is subtracted. $\Rightarrow$ First multiply the brackets. You will notice that this is now a three step equation.
1.	$8x + 12 = 6x - 4$	Expand (multiply) the brackets.
2.	$8x^{-6x} + 12 = 6x^{-6x} - 4$	The opposite to *plus 6x* is *minus 6x*. So subtract $6x$ from both sides so all pronumerals are on the left. This gives $2x + 12 = -4$.
3.	$2x + 12^{-12} = -4^{-12}$	The opposite to *plus 12* is *minus 12*. So subtract 12 from both sides. This gives $2x = -16$.
	$\frac{2x}{2} = \frac{-16}{2}$	The opposite of *times by 2* is *dividing by 2*. So divide both sides by two. This is best done as a fraction
	$x = -8$	This is the solution to the equation. ie For $4(2x + 3) = 6x - 4$ substitute $x = -8$ $4(2 \times -8 + 3) = 6 \times -8 - 4$ $52 = 52$

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Equations involving fractions

Example 11

Step	Solution	Explanations
	$4a + 2 = 2a + 9$	First determine what is happening to the pronumeral. $\Rightarrow$ The pronumeral a is multiplied by 4 and then 2 is added. This is equal to the pronumeral a is multiplied by 2 and then 9 is added. $\Rightarrow$ Reverse this process. ie first get the pronumerals on the left and getting all the numbers on the right. Then undo whatever a is multiplied *by*.
1.	$4a^{-2a} + 2 = 2a^{-2a} + 9$	The opposite to *plus 2a* is *minus 2a*. So subtract $2a$ from both sides so all pronumerals are on the left. This gives $2a + 2 = 9$.
2.	$2a + 2^{-2} = 9^{-2}$	The opposite to *plus 2* is *minus 2*. So subtract 2 from both sides. This gives $2a = 7$.
3.	$\frac{2a}{2} = \frac{7}{2}$	The opposite of *times by 2* is *dividing by 2*. So divide both sides by two. This is best done as a fraction
	$a = 3\frac{1}{2}$	This is the solution to the equation. ie For $5a = 2a + 6$ substitute $a = 3\frac{1}{2}$ $4 \times 3\frac{1}{2} + 2 = 2 \times 3\frac{1}{2} + 9$ $16 = 16$

Example 12

Step	Solution	Explanations
	$2b + 3 = 6b - 5$	First determine what is happening to the pronumeral. $\Rightarrow$ The pronumeral a is multiplied by 4 and then 2 is added. This is equal to the pronumeral a is multiplied by 2 and then 9 is added. $\Rightarrow$ Reverse this process. ie first get the pronumerals on the left and getting all the numbers on the right. Then undo whatever a is multiplied *by*.
1.	$2b^{-6b} + 3 = 6b^{-6b} - 5$	The opposite to *plus 2a* is *minus 2a*. So subtract $2a$ from both sides so all pronumerals are on the left. This gives $-4b - 3 = -5$.
2.	$-4b + 3^{-3} = -5^{-3}$	The opposite to *plus 2* is *minus 2*. So subtract 2 from both sides. This gives $-4b = -8$.
3.	$\frac{-4b}{-4} = \frac{-8}{-4}$	The opposite of *times by 4* is *dividing by 4*. So divide both sides by four. This is best done as a fraction
	$b = 2$	This is the solution to the equation. ie For $2b + 3 = 6b - 5$ substitute $b = 2$ $2 \times 2 + 3 = 6 \times 2 - 5$ $7 = 7$